T.D.C. (Part-II) (Vocational) Examination, 2020

(Subsidiary)

MATHEMATICS

[Paper Code : X(II)-VOC. MATH.(2)]

Time : Three Hours] **[Maximum Marks : 100**

Note : Candidates are required to give their answers in their own words as far as practicable.The questions are of equal value. Answer **any eight** questions. Selecting at least one from each group.

Group-A

1. (a) State and prove Leibnitz theorem.

   (b) If $y = e^{a\sin^{-1}x}$, then prove :

   $$(1-x^2)y_2 - xy_1 - a^2y = 0$$

2. (a) State and prove Euler's theorem on homogeneous function of two variables.

(b) Evaluate :

$$\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$$

3. (a) Show that the sum of intercepts of the tangent to $\sqrt{x}+\sqrt{y}=\sqrt{a}$ upon the co-ordinates is constant.

   (b) Find the Pedal equation of $r^m = a^m \cdot \cos m\theta$

Group-B

4. Integrate **any two** of the following :

   (a) $\int \frac{dx}{1+3e^x+2e^{2x}}$

   (b) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$, where $\beta > \alpha$

   (c) $\int_0^{\pi/2} \frac{dx}{5+4\sin x}$

   (d) $\int \frac{dx}{1+\tan x}$

5. Find the area of the loop of the curve : $x^3 + y^3 = 3axy$.

6. Determine the length of an arc of the cycloid:

$x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, measured from the vertex.

Group-C

7. Solve **any two** of the following :

(a) $ydx - xdy = xydx$

(b) $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

(c) $\cos x \cdot \frac{dy}{dx} + y = \sin x$

8. Solve **any one** of the following :

(a) $y = (1 + p)x + ap^2$

(b) $p^2 - py + x = 0$

(c) $y = px + p - p^2$ where $p = \frac{dy}{dx}$

9 Solve **any two** of the following :

(a) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y\sin 2x$

(b) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}$

(c) $\frac{d^2y}{dx^2} + 9y = x^2$

(d) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 \cdot e^{3x}$

Group-D

10. (a) Define Scalar triple product of vectors. Find its Geometrical meaning.

(b) Prove :

$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$

11. (a) Show that the necessary and sufficient condition for the vector function $\vec{v}$ of scalar variable t to have constant magnitude is $\vec{v} \cdot \frac{d\vec{v}}{dt} = 0$.

(b) If $\vec{r} = \vec{a} \cdot \cos wt + \vec{b} \cdot \sin wt$

then show : $\vec{r} \times \frac{d\vec{r}}{dt} = w\vec{a} \times \vec{b}$

12. (a) Define Divergence and curl of a vertor field.

(b) Prove : $\text{Div.}\left(\vec{a} \pm \vec{b}\right) = \text{div} \cdot \vec{a} \pm \text{div} \cdot \vec{b}$

Group-E

13. (a) Obtain the equations of the line of action of the resultant of a coplanar system of forces acting upon a rigid to body.

(b) Forces P, Q, R act along the sides of the triangle formed by the lines x = 0, y = 0 and $x\cos\theta + y\sin\theta = p$ axes being rectangular. Find the magnitude of the resultant and equation of its line of action.

(a) State and prove the principle of virtual work for a system of coplanar forces.

(b) Five weightless rods of equal lengths are jointed together so as to form a rhombus ABCD with one

diagonal BD. If a weight W be attached to C and the system be suspended from A, show that there is a thrust in BD equal to $\frac{W}{\sqrt{3}}$.

14. Define Simple Harmonic Motion. Prove that the period of a simple Harmonic Motion is independent of amplitude.

15. Find the radial and transverse acceleration of a particle moving in a plane.

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