TDC-Part-I(Voc.) Exam.-2019

Time : 3 hours *Full Marks : 100*

Candidates are required to give their answer in their own words as far as practicable.

The questions are of equal value.

Answer any eight questions, selecting at least one from each group.

Group-A

1. (a) If A,B,C are three sets, Prove that

$$A\times(B-C)=(A\times B)-(A\times C)$$

(b) If $A=\{0,3\}, B=\{4,5\}, C=\{7,12\}$ and $D=\{1,3,5\}$ then show that $(A\times B)\cup(C\times D)=A\cup B\times(B\cup D)$

2. (a) Define types of relation. Give an example of relation which is reflexive and symmetric but not transitive.

(b) Prove that only one-one onto mapping possess an inverse mapping.

3. (a) Define abdian group with example.

(b) If $a,b,c\in G$, Prove that

(i) $ab=ac\Rightarrow b=c$

(ii) $ba=ca\Rightarrow b=c$

4. (a) Define commutative ring with example.

(b) Give an example to show that a ring may not be necessarily an integral domain.

Group-B

5. (a) If $A=\begin{bmatrix}2&3&4\\1&2&3\\-1&1&2\end{bmatrix}$ and $B=\begin{bmatrix}1&3&0\\-1&2&1\\0&0&2\end{bmatrix}$

find AB and BA and show that $AB\neq BA$

(b) Express the following matrix as the sun of symmetric and skew symmetric matrices

$$\begin{bmatrix}2&0&3\\0&3&4\\4&5&0\end{bmatrix}$$

6. (a) Verify that the matrix

$$\frac{1}{3}\begin{bmatrix}1&2&2\\2&1&-2\\-2&2&-1\end{bmatrix}$$ is orthogonal

(b) Find the inverse of the matrix

$$A\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$$

7. (a) Prove that the intersection of two convex sets is also a convex set.

   (b) Prove that any point on the line-Segment joining two points in R^n can be expressed as a convex combination of the two points.

8. Solve the following linear programming problem graphically:

   Max $Z = 5x_1 + 3x_2$

   Subject to $3x_1 + 5x_2 \le 15$

   $5x_1 + 2x_2 \le 10$

   $x_1 x_2 \ge 0$

Group-C

9. (a) Expand tan α in ascending power of α.

   (b) Prove that $\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n = \cos n\theta + i\sin n\theta$

10. (a) If tan(x+iy) = u+iv, Prove that

    $u^2 + v^2 + 2u\cot 2x = 1$

    (b) Prove that $\tan\left\{i\log\frac{a-ib}{a+ib}\right\} = \frac{2ab}{a^2-b^2}$

    where a and b are real quantises.

11. (a) Show that the function $f(x) = x|x|$ is differentiable at the origin.

    (b) State and prove compariso Test (1st Kind) for convergence.

12. Test whether the series whose general term is $u_n = \sqrt{n^4+1} - \sqrt{n^4-1}$ is convergent or divergent.

Group-D

13. (a) Find the condition that the circles

    $x^2 + y^2 + 2g_1x + 2f_1y + C = 0$ and

    $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ intersect orthogonally.

    (b) Show that the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by - c = 0$ cut one another orthogonally.

14. (a) Find the condition that the line y=mx+c may touch the parabola y^2=4ax.

    (b) Prove that the straight line y = mx + c touches the parabola $y^2 = 4a(x + a)$ if $c = ma + \frac{a}{m}$

15. (a) Find the co-ordinates of a point which divides the join of two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the given ratio m : n.

    (b) Find the direction cosines of a line that makes equal angles with the axes.

16. (a) Prove that the first degree equation represents a plane.

    (b) Find the equation of the plane through the points $p(1,1,1)$, Q(3,–1,2) and (–3,5,–4).

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