

2019

INSTRUCTION

Candidates are required to give their answer in their own words as far practicable. Answer any six questions. Select at least one question from each group. All questions carry equal marks.

Group-A

- a) If $y = \sin mx + \cos mx$, prove that $y_n = m^n \{1 + (-1)^n \sin 2mx\}^{\frac{1}{2}}$
b) State and prove Euler's theorem for two variables.
- a) If $y = e^{ax} \sin(bx + c)$ find y_n .
b) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$

- a) Find the asymptotes of the curve $x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$
b) Find the radius of curvature at any point $(a \cos t, b \sin t)$ of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Group-B

- a) Prove that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$
b) Evaluate: $\int_0^{\pi/2} \cos^m x \cos nx dx$
- a) Prove that $\int_0^\infty \frac{\sin mx}{x} dx = \frac{\pi}{2}$
b) Show that the area of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $\frac{3}{8}\pi a^2$

Group-C

- a) Solve any two of the following:
i) $(x+y)^2 \frac{dy}{dx} = a^2$ ii) $\frac{dy}{dx} = \sqrt{y-x}$
iii) $ydx - xdy = xydx$
b) Solve $\frac{dy}{dx} + y \cot x = 2 \cos x$
- a) Solve any two of the following:
i) $(x^3y^2 + xy)^{dx} = dy$ ii) $\frac{dy}{dx} + \frac{y}{x} = x^2$
iii) $p^2 - p(e^x + e^{-x}) + 1 = 0$
- a) Solve: $y = px + \sqrt{a^2p^2 + b^2}$
b) Prove that the system of parabola $y^2 = 4a(x+a)$ is self orthogonal.

Group-D

- a) Find the condition that the two circles cut orthogonally.
b) Define limiting point of circles and find its limiting points. <https://www.brabuonline.com>
- a) Find the standard equation of a parabola.
b) Find the foci, directrix and eccentricity of an ellipse $3x^2 + 4y^2 = 12$

Group-E

- a) Find the angle between two St. Lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) .
b) If a line makes an angle $\alpha, \beta, \gamma, \delta$ with the diagonals of a cube, prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma + \sin^2 \delta = \frac{8}{3}$
- a) Find the equation of a St. Lines in symmetrical form.
b) Show that the lines whose direction Cosines are given by $l+m+n=0$ and $2mn+3nl-5lm=0$ are perpendicular to each other.