Time: 3 Hours | Full Marks: 100

समय – तीन घंटा | अधिकतम अंक – 100

2019

INSTRUCTION

Candidates are required to give their answer in their own words as far practicable. Answer any six questions. Select at least one question from each group. All questions carry equal marks.

Group-A

1. a) Define adjoint and inverse of a matrix.
   b) Find the [illegible] of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 3 & 5 \end{bmatrix}$
2. a) Prove that the product of two orthogonal matrix is orthogonal.
   b) Solve the following simultaneous equations by matrix method:
   $$x + y + z = 6$$
   $$2x + y - 3z = -5$$
   $$3x - 2y + z = 2$$
3. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$

Group-B

4. a) Define cubic and biquadratic equation with example.
   b) Solve the equation $2x^3 - 15x^2 + 37x - 30 = 0$ whose roots are in A.P.
5. Solve the equation $x^3 - 15x^2 - 357x + 5491 = 0$ by Cardon's method.

Group-C

6. a) Define equivalence relation with example.
   b) Define Partition of a set and congruence modulon with example.
7. a) Define abelian additive group with example.
   b) Prove that in a group G the inverse of an element is unique.
8. a) State and prove Euler's theorem.
   b) Define Ring Integral Domain and field.

Group-D

9. a) Find the expansion of Cos $\propto$ by De' Moivre's theorem.
   b) Find all the values of $x^7 - x^4 + x^3 - 1 = 0$
10. a) Prove that $e^{i\theta} = \cos\theta + i \sin$ [illegible]
    b) Prove that [illegible]
    $$(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2(a^2+b^2)^{\frac{m}{2n}} \cos\left\{\frac{m}{n}\tan^{-1}\frac{b}{a}\right\}$$

Group-E

11. a) Define convex set and prove that intersection of two convex sets is also a convex set is also a convex set.
    a) Prove that the set of all feasible solutions of a LPP constitutes a convex set.
12. Solve the following LPP by Simplex method.
    Max $Z = 5x_1 + 7x_2$ subject to constraints
    $$2x_1 + 3x_2 \leq 13$$
    $$3x_1 + 2x_2 \leq 12$$
    $$x_1 \geq 0; x_2 \geq 0$$