2019

Time : 3 hours *Full Marks : 100*

Candidates are required to give their answer in their own words as practicable

The questions are of equal value.

Answer any Six questions, selecting at least one from each group

Group-A

1. (a) State and prove the theorem of perpendicular axes on moment of Inertia.

   (b) Find the moment of inertia of a triangle ABC about a perpendicular to a plane through A.

2. (a) Show that the moments and products of inertia of a uniform triangle about any lines are the same as the moments and product of inertia about the same lines, of three particles placed at the middle points of the sides, each equal to one-third of the mass of the triangle.

   (b) Show that the equation of the momental ellipsoid at the corner of a cube of side 2a referred to its principal axes is

P.T.O.

$$2x^2 + 11(y^2 + z^2) = c$$

3. (a) Define centre of suspension and centre of oscillation. Prove that centre of suspension and centre of oscillation are convertable.

   (b) A pendulum is supported at O, and P is the centre of oscillation. Show that, it an additional weight is rigidly attached at P, the period of oscillation is un altered.

4. (a) Deduce the general equation of motion of a rigid body from D'Alemberts principle

   (b) Find the kinetic energy of a rigid body rotating about a fixed point.

Group-B

5. (a) Find the attraction of a then uniform spherical shell on an internal point P.

   (b) Find the potential of a finite rod AB of small cross section K and uniform density e at any external point P.

6. (a) State and prove Poisson's theorem.

   (b) Show that the system of co-axial cylinders.

$$x^2 + y^2 + 2\lambda x + c^2 = o$$

Can form a system of equipotential surfaces. Also find the law of potential.

7. (a) Find the pressure at a point in a heavy homogeneous liquid at rest under gravity.

(b) Find the centre of Pressure of a rectangle immersed in a homogeneous liquid with one side in the surface.

8. (a) Find the condition for equilibrium of a body floating in two liquids.

(b) A conical Vessel, of height h and vertical angle 2α, contains water whose volume is one half that of the cone ? If the vessel and the contained water revolve with angular velocity W, and no water over flows. Show that W must not be greater than

$$\sqrt{\frac{29}{3h}\cot\alpha}$$

Group-C

9. (a) Solve any one of the following

(i) $x\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^x$

(ii) $x\frac{d^2y}{dx^2} - (x^2+2x)\frac{dy}{dx} + (x+2)y = x^3e^x$

(b) Solve by method of Variation of Parameters.

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

10. Solve

(i) $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$

(ii) $(y^2+yz+z^2)dx + (z^2+zx+x^2)dy + (x^2+xy+y^2)dz = 0$

11. Solve any two, using charpit's method

(i) $px + qy = pq$

(ii) $(p^2+q^2)y = qz$

(iii) $z - px - qy = p^2+q^2$

12. (a) Explain Monge's method for solving the equation.

$$Rr + Ss + Tt = V$$

(b) Solve $r = a^2t$

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