2019

Time : 3 hours *Full Marks : 100*

Candidates are required to give their answers in their own words as far as practicable.

the questions are of equal value

Answer any Six questions, selecting at least one from each group.

Group-A

1. (a) Define normal sub group. Prove that every sub group of an abelion group is normal

   (b) State and prove class equation for a finite group G.

2. (a) State and prove fundamental theorem on homomorphism of groups.

   (b) If f is a homomorphism of a group G into a group G' with kernel K, then prove that K is a normal sub group of G.

3. Prove that a group G is the direct product of its two sub groups H and K if and only if.

P.T.O.

   (i) H and K are normal sub groups of G.

   (ii) $H \cap K = \{e\}$

   (iii) G = HK

4. State and prove first Sylow theorem.

Group-B

5. (a) Prove that every finite integral domain is a field.

   (b) Give an example of a skew-field which is not a field.

6. (a) Define sub-ring. Prove that intersection of two sub rings is a sub ring.

   (b) Define characteristic of a ring.

   Prove that the characteristic of an integral domain is 0 or n > 0 according as the order of any non-zero element regarded as a member of additive group of integral domain is either 0 or n.

7. Prove that the set R [x] of all polynomials over an arbitrary ring R is a ring with respect to addition and multiplication of polynomials.

8. (a) Prove that a field has no proper ideals.

   (b) Prove that a commutative ring with unity is a field if it has no proper devisors.

Group-C

9. (a) Define vector space. Prove that a field K can be regarded as a vector space over any sub field F of K.

   (b) Prove that the necessary and sufficient condition for a non-empty sub set W of a vector space V (F) to be a sub-space of V is that W is closed under vector addition and scalar multiplication in V.

10. (a) Prove that the intersection of any two sub spaces W_1 and W_2 of a vector space V (F) is also a sub space of V (F) but their union may not be a subspace of V (F).

    (b) Determine whether or not the following vectors form a basis of R^3 :

$$(1,1,2), (1,2,5), (5,3,4)$$

11. Define liner transformation T of a vector space U to a vector space V over F.

    It dim. (U) = n and rank of T is = γ

    then prove : nullity of T = $n - \gamma$

http://www.brabuonline.com

12. Find the eigen values and corresponding eigen vectors of the matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$$

***