2019

Time : 3 Hrs ***Full Marks : 100***

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any six questions selecting at least one from each group.

Group A

1. (a) State and prove Moore-Osgood theorem.

   (b) Discuss the continuity of

$$f(x,y) = 2xy\left(\frac{x^2-y^2}{x^2+y^2}\right),\ (x,y)\neq(o,o)$$

$$f(o,o)=o$$

at the origin

2. (a) State and prove Young's theorem.

   (b) Let the function f be defined by

$$f(x,y)=\begin{cases}\dfrac{x^2y^2}{x^2+y^2}, & x^2+y^2\neq 0\\ 0. & other\ \ wise\end{cases}$$

Show that $f_{xy}(o,o)=f_{yx}(o,o)$ although f_{yx} is continuous at (o,o).

3. State and prove Implicit function theorem.

4. Discuss Lagrange's method of undetermined multipliers.

Group B

5. (a) State and prove Darbox theorem.

   (b) Let f be a bounded function defined over [a,b] (where a < b) by

$$f(x)=\begin{cases}1, where\ \ x\ \ is\ \ rational\\ -1, where\ \ x\ \ is\ \ irrational\end{cases}$$

   Prove that f is not Riemonn integrable but $|f|$ is Riemonn integrable

6. (a) Prove that a function continuous on an inter val $[a,b]$ is R-integrable on $[a,b]$. Show by an example that the converse is not true.

   (b) Show that the function

$$f(x)=\frac{1}{2^n},\ \text{where}\ \frac{1}{2^{n+1}}\le x\le\frac{1}{2^n}(n=0,1,2,...)$$

$$f(o)=0$$

   is R-integrable in $[0,1]$. Also find its R- integral.

7. (a) Define Riemann-Stieltjes integral. Prove that if f is monotonic in $[a,b]$ and α is continuous on $[a,b]$ then show that $f\in R(\alpha)$ on $[a,b]$

(b) If $f \in R(\alpha)$ and $f \in R(\alpha)$ on $[a,b]$ then prove $f.g \in R(\alpha)$

8. (a) Define Beta function and Gamma function prove that $B(m,n)$ is symmetric with respect to m and n.

(b) Discuss the convergence of the integral

$$\int_0^\infty e^{-x}.x^{n-1}dx$$

Group C

9. (a) State and prove Cauchy's general principle of uniform convergence of a sequence $\{f_n\}$ of function.

(b) Show that the series

$$\sum \frac{x}{n^p + x^2 n^g} \text{ is uniformly converges}$$

Over any bounded interval $[a,b]$ for

(i) $g > 0,\ p > 1$ and $0 < p \le 1$, $p + q > 2$

10. (a) State and prove Abel's test.

(b) Show that the series $\sum \frac{Cosnx}{n^2}$ converges uniformly on the real line.

11. (a) State and prove Dini's Test.

(b) Let $\{f_n\}$ be a sequence of real valued fu[redacted] a metric space (x,d) which converges uniformi the function f on x. If each $f_n(n-1,2,..........)$ is continuous on x, then prove that f is also continuous on x.

12. (a) State and prove term by term differentiation theorem on uniform convergence of a series.

(b) Show that the series for which

$$f_n(x) = \frac{nx}{1+n^2x^2}, 0 \le x \le 1$$

can be differentiated term by term at x=0

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